ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS INC.

Total Time: 25 minutes

Total Marks: 19 marks

Specialist Mathematics Units 3&4

Topic Test 2

(Wed, May 11th)

Resource Free

ClassPad Calculators are NOT permitted.
Miscellaneous Formulae Sheet is permitted.

Name: _____

1. [1 & 2 = 3 marks]

Points A, B and C are such that $\overrightarrow{AB} = 2\underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{j}} - \underset{\sim}{\mathbf{k}}$ and $\overrightarrow{AC} = -\underset{\sim}{\mathbf{i}} + 2\underset{\sim}{\mathbf{j}} + 2\underset{\sim}{\mathbf{k}}$.

(a) Find a vector that is perpendicular to vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Point A has position vector $3\underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{j}} - 2\underset{\sim}{\mathbf{k}}$.

(i) Find the vector equation of the plane that contains points A, B and C.

(ii) Find the position vector of point B.

2. [1, 1, 1, 1 & 2 = 6 marks]

Consider the two points, *A* and B, with position vectors $\underset{\sim}{\boldsymbol{a}} = \underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}} + 2\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{b}} = 2\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 4\underset{\sim}{\boldsymbol{k}}$.

(a) Find $2\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}}$

(b) Find the magnitude of $\underset{\sim}{\mathbf{a}}$.

(c) Vector $\underset{\sim}{\boldsymbol{f}}$ is in the direction of $\underset{\sim}{\boldsymbol{a}}$ and has magnitude of 5. Find vector $\underset{\sim}{\boldsymbol{f}}$.

(d) Find the vector $\overrightarrow{AB}$.

(e) Find the position vector of the point *P* that divides *AB* internally in the ratio of 2:3.
i.e. AP:PB = 2:3.

3. [4 marks]

Find the exact shortest distance between the line with vector equation $\underset{\sim}{\boldsymbol{r}}(\lambda) = \begin{pmatrix} \lambda \\ 3+2\lambda \end{pmatrix}$ and the point A with position vector $\underset{\sim}{\boldsymbol{i}} + 10\underset{\sim}{\boldsymbol{j}}$.

4. [1, 2 & 3 = 6 marks]

Two particles are moving on paths described by the vector equations $\underset{\sim}{\mathbf{r}}_A = (3t-1)\underset{\sim}{\mathbf{i}} + 5t\underset{\sim}{\mathbf{j}}$ and $\underset{\sim}{\mathbf{r}}_B = (2t+5)\underset{\sim}{\mathbf{i}} + (t^2-6)\underset{\sim}{\mathbf{j}}$ respectively.

(a) Find the Cartesian equation of the path of particle B.

(b) Use vector methods to find the exact distance between the two particles when $t = 4$.

(c) Use vector methods to prove that the particles collide and find the time and position vector of the point of collision.

End of Resource Free

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Total Time: 25 minutes

Total Marks: 21 marks

Specialist Mathematics Units 3&4

Topic Test 2

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Miscellaneous Formulae Sheet, half an A4 size page of notes and ClassPad Calculators are permitted.

Name: _____

In this section of the test, you should use your ClassPad calculator. You **must** show appropriate mathematics so that your method is clear. Do not write ClassPad instructions in your method. Write only appropriate mathematical notation.

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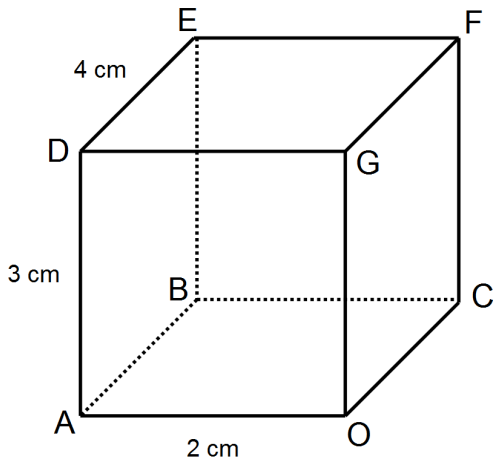
5. [2 marks]

Points A and B have position vectors $4\underset{\sim}{\mathbf{i}} - 5\underset{\sim}{\mathbf{j}} - 2k$ and $8\underset{\sim}{\mathbf{i}} - 5\underset{\sim}{\mathbf{j}} + 6\underset{\sim}{\mathbf{k}}$ respectively relative to an origin O.
Use vector methods to find the area of ΔAOB.

6. [2, 2 & 2 = 6 marks]

The rectangular prism to the right has base OABC and top of GDEF with G, D, E and F above O, A, B and C respectively.

DE = 4 cm, DA = 3 cm and AO = 2 cm and let the origin be point O.



The coordinates of G relative to the origin are (0, 0, 3)

(a) State the position vector of point

(i) D

(ii) P, the midpoint of BC

(b) Find a vector equation of the line through points P and D.

(c) Use vector methods to find the angle the line through P and D makes with the base OABC.

7. [1 & 3 = 4 marks]

At 10 am, particle B leaves point P, position vector $2\underset{\sim}{\boldsymbol{i}} + 5\underset{\sim}{\boldsymbol{j}}$ metres relative to origin O, with velocity $-\underset{\sim}{\boldsymbol{i}} + 5\underset{\sim}{\boldsymbol{j}}$ m/s.

(a) Find the position of B relative to the origin after 5 seconds.

(b) Find the amount of time after 10 am that it takes for B to first be 10 metres from the point Q which has position vector $3\underset{\sim}{\boldsymbol{i}} + 46\underset{\sim}{\boldsymbol{j}}$.

8. [1 & 3 = 4 marks]

(a) Find the angle between the planes $\underset{\sim}{\mathbf{r}} \cdot \left(2\underset{\sim}{\mathbf{i}} - \underset{\sim}{\mathbf{j}} - 3\underset{\sim}{\mathbf{k}}\right) = 10$ and $x + 3y - 2z = 16$.

(b) Find, in scalar product form, the vector equation of the plane $\underset{\sim}{\mathbf{r}} = (1 + 3\lambda + 2\mu)\underset{\sim}{\mathbf{i}} + (1 + \lambda + 4\mu)\underset{\sim}{\mathbf{j}} - \mu\underset{\sim}{\mathbf{k}}$.

9. [2 marks]

The points $A(4,-2,3)$, $B(-1,3,4)$, $C(2,4,-2)$ and $D(7,-1,-3)$ form quadrilateral ABCD.

Use vector methods to prove that ABCD is a parallelogram.

10. [3 marks]

The position vectors of three non-collinear points A, B and C, with respect to an origin O, are $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\mathbf{c}}$ respectively.

Given that O does not lie in the plane ABC, show that $\alpha + \beta + \gamma = 1$ if the point Q with position vector $\underset{\sim}{\boldsymbol{q}} = \alpha\underset{\sim}{\mathbf{a}} + \beta\underset{\sim}{\mathbf{b}} + \gamma\underset{\sim}{\mathbf{c}}$ lies in the plane ABC.

End of Assessment